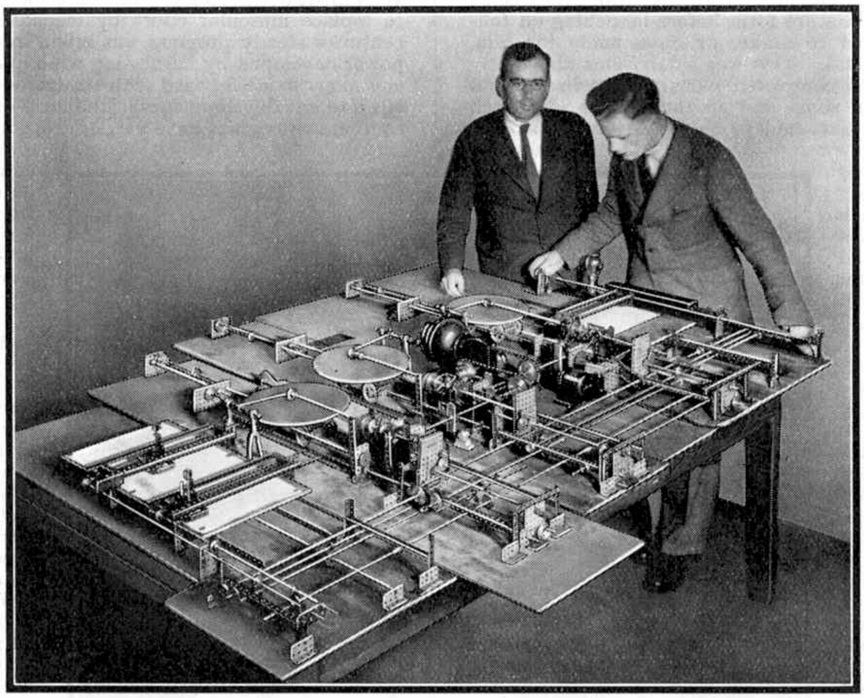
Machine Solves Mathematical Problems

A Wonderful Meccano Mechanism

FROM time to time examples have been given in the "*M.M.*" of the readiness with which the most complicated mechanisms can be reproduced in Meccano. An excellent instance of this is the wonderful astronomical clock described on page 170 of our issue for March, 1933, which automatically gives a wealth of interesting and useful astronomical information. More recently Meccano has been used in the construction of a remarkable machine that solves in a few minutes complicated equations that otherwise could only be dealt with by laborious calculations occupying many hours. The original of this model is a machine known as the Differential Analyser that was developed by Dr. V. Bush, Vice-President of the Massachusetts Institute of Technology, Cambridge, U.S.A. In constructing this machine, which at present is the only one of its kind in the world, Dr. Bush's purpose was to shorten the labour of making calculations from the complicated equations met with in working out problems in electrical and other branches of engineering, and also in physics and astronomy.

The solution of these equations is often difficult, and the kind of arithmetical work involved is not well known, and is prolonged and wearisome. Further, human calculators are liable to error, especially in carrying out long series of similar calculations such as are often necessary in work of this kind. These difficulties are avoided by the use of the machine. In a few hours it can be set to provide solutions for equations of astonishing complexity, and then accurate results can be obtained from it in a convenient form in a few minutes.



Professor D. R. Hartree and Mr. A. Porter, of the Department of Mathematics, The University, Manchester, with a wonderful Meccano mechanism they have constructed to solve complex mathematical problems. This mechanism is a reproduction on a smaller scale of the Bush Differential Analyser illustrated on the opposite page, and a simpler form of it is illustrated at the top of page 444.

A general view of the Differential Analyser is given in the upper illustration on the opposite page. It has been described as one of the most comprehensive pieces of mathematical machinery ever built, but in spite of its formidable appearance it is really simple in construction. It consists of an assembly of units that mechanically add, subtract, and carry out other and more complicated mathematical operations, and by adding more units it can readily be enlarged to deal with problems of increasing complexity. As a matter of fact it grows so continuously that its designer has expressed the opinion that it will never really be complete.

The most important mathematical operation that the machine carries out distinguishes it from other kinds of calculating machine, and makes it unique in the range and complexity of problems to which it can be applied. This operation can best be explained by an example. Suppose that a motor car is starting from rest, and that we have a record of its speed at each moment from the start. This record might be in the form of a graph showing how the speed varied with the time from the start; in handling the problem by the Differential Analyser the information actually would be supplied to it in the form of such a graph. From this information we require to know how far the car goes in, say, two minutes. We can find this approximately by dividing the period of two minutes into smaller intervals, for example into 12 intervals of 10 seconds each; and by imagining that the speed remains constant in each interval, then suddenly changes to another constant value in the next interval, and so on. Thus we can find the distance travelled in each period by multiplying each time interval by the supposed constant speed corresponding to it, and finally add up the distances travelled in successive intervals to find the total distance covered.

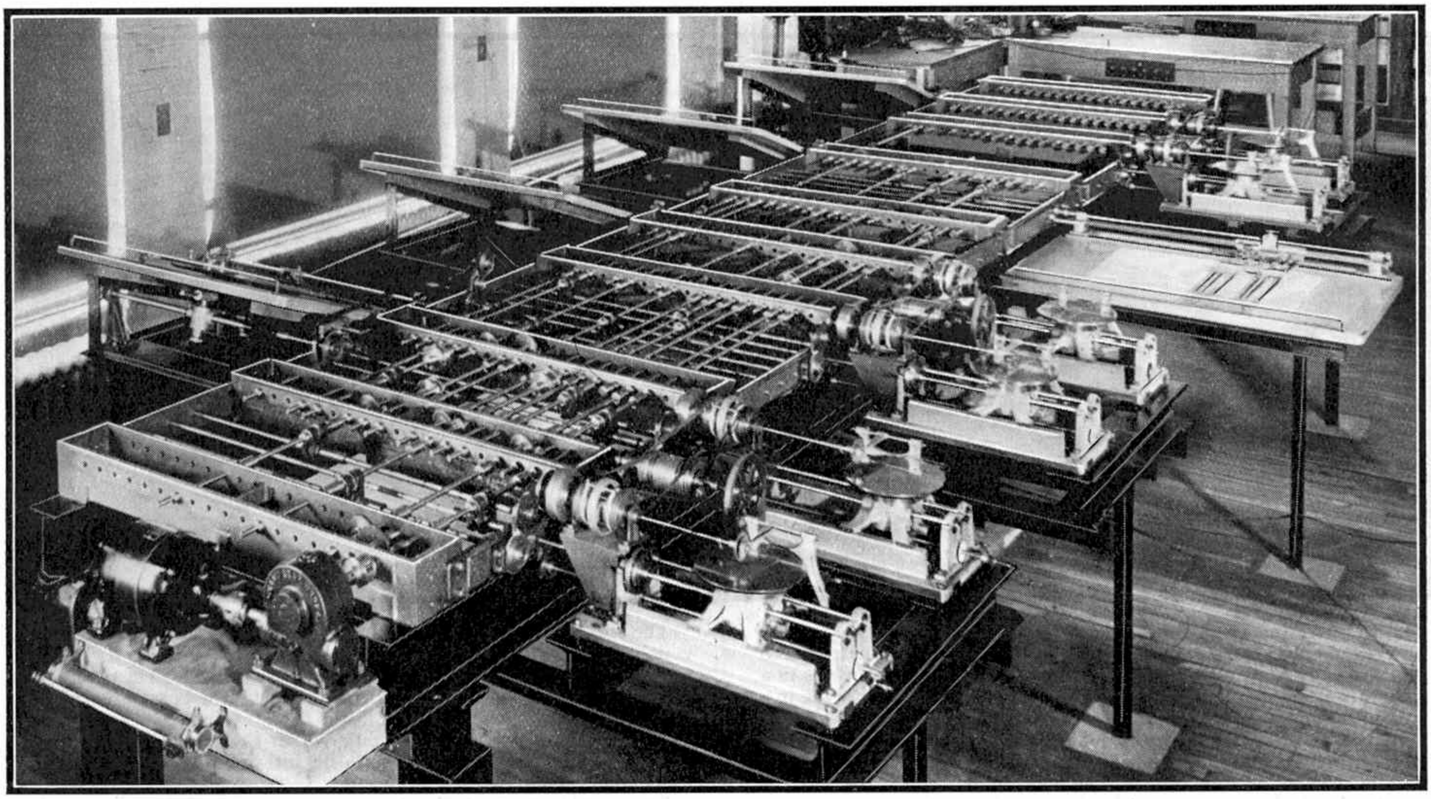
The result will be only approximate, because the speed actually is not constant in each interval, as we have imagined it to be. The error on this account can be decreased, however, by dividing up the period into smaller intervals, say 24 of five seconds each, or 60 of two seconds each, etc., until the variation of speed in each interval becomes too small to matter. By taking small enough intervals an accurate result can be calculated, however rapidly the speed varies during the total period concerned.

The mathematical operation in which the distance traversed is derived from the speed, which is regarded as known, is technically called "integration"; and the essential feature of the machine is that it incorporates devices called "integrators" for carrying out this operation mechanically. How an integrator works will be described later.

This operation of integration arises in the working out of the most varied problems, in astronomy, physics, chemistry, and engineering, and the scope of problems that can be investigated by the machine is correspondingly wide.

In the centre of the machine is a set of longitudinal shafts, which in our illustration can be seen running from the lower left-hand corner towards the right-hand upper corner. These shafts can be geared to each other so as to rotate at various relative speeds, and the rate at which each turns represents a term in the equation for which a solution is required. The manner in which they are geared depends on the relation between the terms. For instance, if any two terms are to be added together, the shafts representing them are connected with a third by means of differential gearing designed to make the third shaft turn at a speed representing the sum of the speeds of the shafts driving it. More complicated relationships are worked out through special devices such as the integrators already mentioned, which can be seen on the right of the longitudinal shafts; and others known as input tables, which are on the left. Both devices are driven by means of cross shafts.

When the necessary connections have been made, one of the shafts is driven by an electric motor, and in turn drives the other shafts, each at its appropriate speed. When this is done, the speed of the shaft representing the term of which the value is to be found then gives the required solution. For the type of equation dealt with on the machine, the kind of result most usually required is not a single number, but a series of related numbers. For example, in the case of the motor car already considered we wished to know the distance the car travelled in two minutes. To complete our information, however, we require to know how far the car goes in three, four, five, or any other number of seconds. The machine

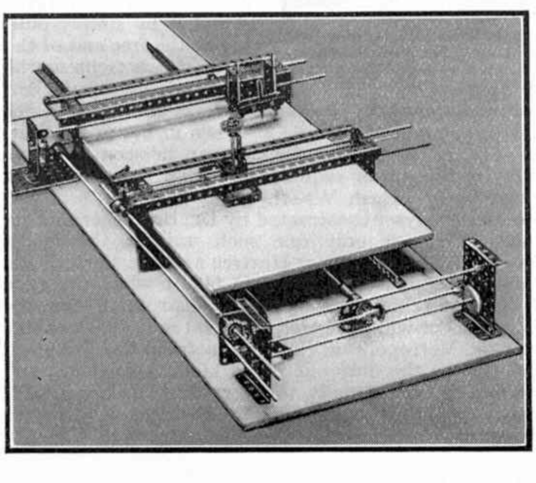


The Bush Differential Analyser at the Massachusetts Institute of Technology, Cambridge, U.S.A. This machine is illustrated also on page 441. We are indebted to the courtesy of Dr. V. Bush for our photographs. Below are the output (left) and input tables of the Meccano reproduction of the machine described in this article.

provides this solution in a very ingenious manner, for the shaft representing distance travelled—the solution of the equation—is made to drive through cross shafts a pencil that moves across a sheet of paper pinned to a board, and this itself is moved at right angles through similar gearing connected to the longitudinal shaft representing time. Thus the pencil draws a curve that shows how the distance travelled increases with the time, and this is the solution to the equation.

The board on which this curve is drawn is called the output table, and in our photograph of the machine is shown on the right of the horizontal shafts. If the pencil were driven from a shaft representing the distance travelled by the car, the curve drawn then would enable the distance at any instant to be found, and the machine can readily be adjusted to give solutions in alternative forms as may be desired. In practice curves for two quantities concerned in the solution can be drawn by using two pencil carriages.

The Meccano model of this ingenious machine was designed and constructed by Professor D. R. Hartree, F.R.S., of the Department of Mathematics, The University, Manchester, and Mr. A. Porter, and is shown in the upper illustration on page 444. It was built for the purpose of demonstrating the principles on which the Bush Differential Analyser works, but it turned out to be more than a demonstration model, and to be capable of solving many equations with a considerable degree of accuracy. It is simpler than the original, but has just the same flexibility, and its range of operations can be increased by the addition of new units. Professor Hartree is finding it of great value in connection with his researches on electrical problems connected with the constitution of the atom, and thus Meccano is playing an important part in an interesting field of scientific work.

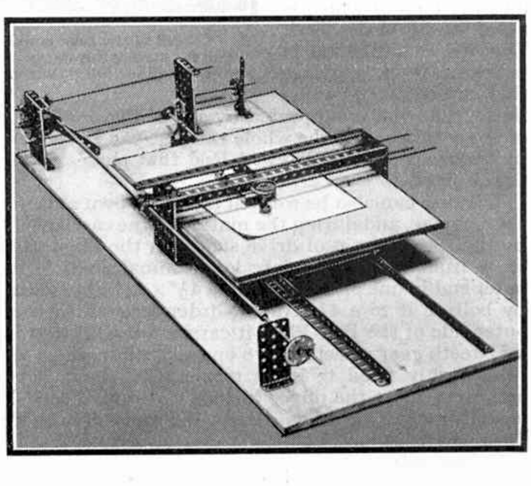


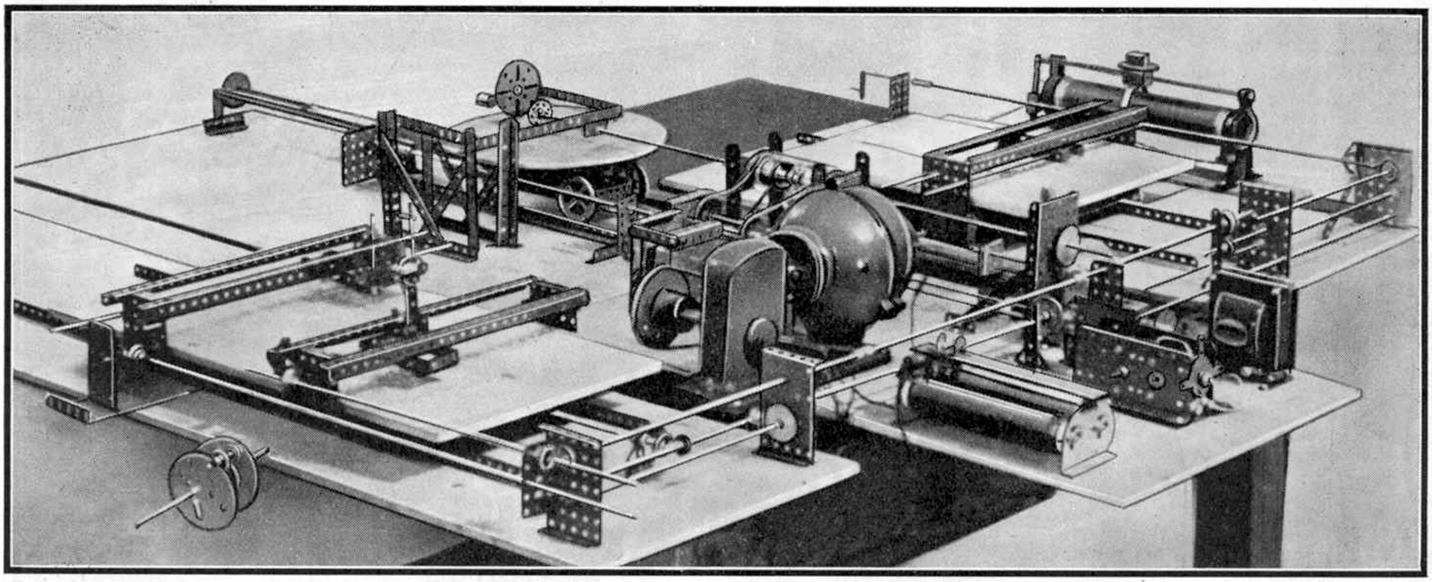
As will be seen from the illustration on page 444, the general layout of Professor Hartree's model is similar to that of the Bush machine. The shafts consist of Axle Rods joined where necessary by means of Couplings in order to provide the necessary length. These shafts are journalled in holes in suitable positions in $3\frac{1}{2}'' \times 2\frac{1}{2}''$ Flanged Plates screwed to the table, and drive the cross shafts, also consisting of Axle Rods, by means of $\frac{7}{8}''$ Bevel Gears. A 6-volt Meccano Electric Motor is employed to drive the shafts, and this, together with its transformer, can be seen on the right of the photograph.

The output table of the machine is shown in detail in the left-hand lower corner of this page. The pencil that draws the output curve is supported by a carriage, built of Meccano parts, that moves along two Axle Rods that support it and are fitted in a framework of Girders bolted to $3\frac{1}{2}'' \times 2\frac{1}{2}''$ Flanged Plates screwed down to the table. This is moved by the rotation of a Screwed Rod that passes through a Threaded Crank fixed to the carriage, and is fitted with Collars where it passes through the Flanged Plates so that it cannot move along its length. The Screwed Rod is driven by $\frac{7}{8}''$ Bevel Gears from a cross shaft, which is similarly driven from the main shafts of the machine.

The output table is moved at right angles to the direction followed by the pencil. The drive from the appropriate main shaft is transmitted to it by a Screwed Rod passing under it through a Threaded Crank bolted to an Angle Girder screwed to the under side of the table. A second pencil carriage is provided, as in the original machine. The additional cross shaft can be seen on the left, below the first one.

The input table of the machine is shown in the right-hand lower corner of this page, together with a part of the longitudinal shaft system that has been disconnected from the rest of the machine for convenience in obtaining a photograph. Its purpose is to enable two shafts connected

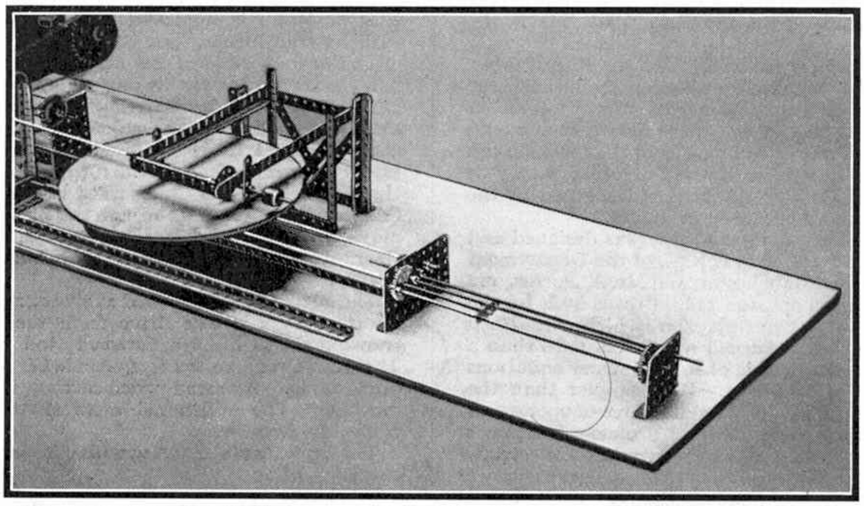




to it to be rotated at speeds that vary continuously relative to each other. To enable this to be done, a curve to represent the required relation is drawn on paper pinned to the table, which moves in a direction at right angles to the longitudinal shafts, and is driven in a similar manner to the output table. A pointer on a moving carriage similar to those already described is kept as nearly as possible on the curve by turning the Bush Wheel, provided with a Threaded Pin as a handle, shown on the right. The motion is transmitted to the pointer by $\frac{7}{8}''$ Bevel Gears and a length of Screwed Rod, and the appropriate speed is communicated to one of the shafts in the main assembly to which it is geared. Thus the two longitudinal shafts concerned are rotated at their correct relative speeds when the machine is in operation.

In solving the complicated equations for which the machine is designed it is necessary to connect longitudinal shafts together in a more complex manner than the use of an input table allows. Three shafts are concerned in these cases, the rate of rotation of one depending on that of the second and on the total rotation of the third. The connection is established by means of the integrator. This is the round horizontal disc shown in the illustration of the machine, and in the lower illustration on this page, on the upper surface of which rolls a wheel that can rotate about a horizontal axis.

To the under side of the horizontal disc is screwed a Bush Wheel, into the boss of which is fitted a 3″ Axle Rod. This rod is journalled in bearings in a carriage that can be moved along rails. The disc is supported by a Rail Bearing that also forms part of the carriage, and the whole carriage can be moved along the rails by the rotation of a Screwed Rod that passes through a Threaded Crank fixed to the carriage.



At the head of the page is the Meccano reproduction of the Differential Analyser in its earliest form, from which the mechanism shown on page 442 was developed by the addition of new units. The lower illustration shows one of the integrators of the model. These photographs and those at the foot of page 443 are reproduced by courtesy of Professor D. R. Hartree.

The disc can also be rotated about its own axis in any position of the carriage, and during the motion of the carriage; this is achieved by the special form of drive shown in the illustrations. The cross shaft from the appropriate longitudinal shaft is journalled at its outer end in one of the holes in a $4\frac{1}{2}'' \times 2\frac{1}{2}''$ Flat Plate held vertically by bolting it to a $4\frac{1}{2}''$ Angle Girder screwed in the table. On the outer side of the Flat Plate it carries a pinion that indirectly drives a 57-teeth gear wheel, in two opposite outer holes of which are Axle Rods firmly fixed to it by means of Collars. There is a similar arrangement at the opposite ends of the two Axle Rods, giving an assembly that turns as a unit in obedience to the drive of the cross shaft.

The two Axle Rods pass through the holes of a Double Arm Crank, which of course turns with them. The shaft that actually drives the rotating table is secured to the boss of the Double Arm Crank by means of a set screw, and passes freely through the boss of one of the 57-teeth Gear Wheels, as shown in the illustration. As the position of the carriage changes, the Double Arm Crank moves along the two Axle Rods, and thus there is no interruption to the turning movement given to it.

The combination of the two movements described has to be communicated to one of the horizontal shafts. This is done by means of the Bush Wheel that rests on the rotating table, and is turned by it. The speed at which the Bush Wheel turns depends on the rate at which the disc is turning, and also on its distance from the centre of the table. The actual position of the Bush Wheel is fixed, so that if the disc is turning at a constant rate, and the carriage on which it is mounted is moving, the speed of rotation of the Bush Wheel varies continuously, and this rotation is communicated to one of the longitudinal shafts of the machine by means of a cross shaft and Bevel Gears.

The torque, or turning power, of the Bush Wheel is very small; and in itself is far from sufficient to drive the longitudinal shaft to which it is connected. Its effect therefore is multiplied by means of a special device that acts on the capstan principle. In this a rotating pulley is made to grip a rope given two or three turns round it by simply pulling at the free end of the rope, which tightens the turns and gives sufficient grip for the turning power of the capstan to take effect. The device that applies this principle in the model is shown in the upper illustration on this page, in the centre of which can be seen the large motor that provides the power required to drive the cross shaft that is controlled by the Bush Wheel.

In the Differential Analyser constructed by Dr. Bush there are six integrating units. There is only one such unit in the model described in this article, but Professor Hartree and Mr. Porter have constructed a new model, illustrated on page 442, in which there are three. A full-sized machine now under construction for the University of Manchester will have eight integrators. This machine is being built by the Metropolitan Vickers Electrical Co. Ltd., and when completed will be the only one of its kind in England, and one of the first two, if not actually the very first, in Europe.

We are greatly indebted to Professor Hartree, F.R.S., for valuable assistance in the preparation of this article.